A Comparison of Four Solution Methods for the Analysis of a Trapezoidal Fin

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A comparison is made of the temperature profile and the heat loss from a trapezoidal fin using four methods. These four methods are the one- and two-dimensional analytical, the two-dimensional finite difference and a two-dimensional modified finite difference method. The two-dimensional analytical method was arbitrarily chosen as the reference. The non-dimensional fin length is restricted to be less than 2 to prevent errors which might occur due to large values of Δx in the finite difference methods. The values of the Biot number range from 0.01 to 1.0 while the thermal conductivity of the fin and fin's convection coefficients are assumed constant. The results show that (1) in the view of the heat loss from the trapezoidal fin, all four methods can be used to obtain the solutions within 3% with each other for the given range of Biot number and the non-dimensional fin length, (2) for the non-dimensional temperature, the one-dimensional analytical method does not produce good results as compared to the other three methods when the Biot number is 1.0, and (3) by using a two-dimensional modified finite difference method instead of the two-dimensional finite difference method, the relative difference in the heat loss as compared to a two-dimensional analytical method is reduced considerably.

Key Words: Trapezoidal Fin, Analytical Method, Finite Difference Method, Heat Loss

Nomenclature

Bi : Trapezoidal fin Biot number $(=hl/k)$
I, K : Modified Bessel functions
h : Fin convection coefficientk
k : Thermal conductivity
l : One half fin thickness at the root
L′ : Fin length-base to tip dimension
L : Non-dimensional fin length $(=L'/l)$
Q_{1A} : Calculated heat loss using the one-dimensional analytical method
Q_{2A} : Calculated heat loss using the two-dimensional analytical method
Q_{2F} : Calculated heat loss using the two-dimensional finite difference method
Q_{MF} : Calculated heat loss using the two-dimensional modified finite difference method
T : Fin temperature
T_w : Fin base temperature
T_∞ : Ambient temperature
x' : Horizontal variable $x' \leq L'$ for a 2-D analysis and $L' \leq x' \leq 2L'$ for the 1-D analysis
x : $x'/l \leq L$ for a 2-D analysis and $L \leq x \leq 2L$ for the 1-D analysis
y' : Across the fin variable $y' \leq |+l|$
y : $y'/l \leq |1|$
Δx : Increment of x along the horizontal (base to tip)
Δy : Increment of y across the fin at the wall
r : $\Delta x/\Delta y$

Greek characters

θ : Non-dimensional fin temperature $(T-T_\infty)/(T_w-T_\infty)$
θ_{1A} : θ calculated using the one-dimensional analytical method
θ_{2A} : θ calculated using the two-dimensional analytical method
θ_{2F} : θ calculated using the two-dimensional

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finite difference method

θ_{MF} : θ calculated using the two-dimensional modified finite difference method

λ_n : Eigenvalues ($n=1, 2, 3, \cdots\cdots$)

1. Introduction

Finned surfaces are used in many applications. For instance, they are vital parts of the cylinder cases in an air-craft engine, compact heat exchanger, and other heat transfer machinery. In order to analyze the performance of a fin, two general classes of solution methods have been used. The finite difference and finite element methods (Meric, 1979; Hyrmak and McRae, 1985; Hensel and Hills, 1986; Muralidhar, 1990; Kang and Look, 1993), requiring nodal or element descriptions on and within the boundary of the model, are one class of analyses while another class is analytical methods (Lau and Tan, 1963; Irey, 1968; Look, 1988; Ma, Behbahani, and Tsuei, 1991; Kang and Look, 1993; Kang, 1997) which require descriptions on the boundary of the fin. Usually only one or two of these methods are used to analyze a particular fin. Unfortunately each method has its own error which occurs maturally.

In this paper, four different methods are used to analyze the performance of a trapezoidal fin. In particular, comparisons of the heat loss from the fin are made, as a function of the non-dimensional fin length and the Biot number, as well as the temperature variation along the fin center line and the upper fin surface.

The first method presented in this paper is the one-dimensional analytical method (Avrami and Little, 1942; Lau and Tan, 1963; Irey, 1968; Snider and Kraus, 1983). The second method is a two-dimensional analytical method (Buccini and Soliman, 1986; Look, 1988; Ma, Behbahani, and Tsuei, 1991; Kang and Look, 1993; Kang, 1997), which uses the separation of variable technique and the orthogonal principle. A two-dimensional finite difference approach, which was used in Kang and Look (1993) and was based on a direct application of the first law of thermodynamics, is the third method discussed. Finally, a two-dimensional modified finite difference method is presented in which the nodes near the sloped lateral surface of the fin are treated differently than the interior point. In order to compare the results of these four methods using a relative difference scheme, the two-dimensional analytical results were chosen as the reference because it is the classical method (Reiser and Appl, 1974) which is usually used to solve the governing differential equations (separation of variables which satisfy the boundary conditions for multi-dimensional analysis).

For the finite difference methods, 40 nodes in the upper half of the trapezoidal fin was sufficient. That is, test run for several different configurations (number of nodes) indicated that 40 nodes were sufficient for the solutions to converge. Further, the non-dimensional fin length was restricted to be less than 2 in order to prevent any error which might result due to Δx being too large as the non-dimensional fin length increased. Each analysis was based on the following assumptions: the base temperature, surrounding convection coefficient and the fin thermal conductivity are fixed and the condition is steady-state.

2. Analyses

2.1 One-dimensional analytical (1A)

Based upon the fin geometry illustrated in Fig. 1, the one dimensional governing differential equation is presented as Eq. (1).

$$\frac{d^2\theta}{dx^2}+\frac{1}{x}\frac{d\theta}{dx}-Bi\cdot\sqrt{(1+4L^2)}\cdot\frac{\theta}{x}=0 \quad (1)$$

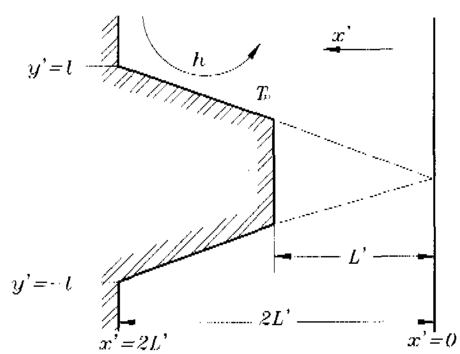

Fig. 1 Trapezoidal fin geometry used in the one-dimensional analysis.

Two boundary conditions for this approach are

$$\theta=1 \qquad \text{at } x=2L \qquad (2)$$

$$\frac{d\theta}{dx}-Bi\cdot\theta=0 \qquad \text{at } x=L \qquad (3)$$

The solution to Eqs. (1)-(3) are given as Eqs. (4)-(7):

$$\theta_{1A}(x)=\frac{C_1\cdot I_0(2p\sqrt{x})+C_2\cdot K_0(2p\sqrt{x})}{C_1\cdot I_0(2p\sqrt{2L})+C_2\cdot K_0(2p\sqrt{2L})} \qquad (4)$$

where

$$p=\sqrt{Bi}\cdot(1+4L^2)^{(1/4)} \qquad (5)$$

$$C_1=\frac{p}{\sqrt{L}}\cdot K_1(2p\sqrt{L})+Bi\cdot K_0(2p\sqrt{L}) \qquad (6)$$

$$C_2=\frac{p}{\sqrt{L}}\cdot I_1(2p\sqrt{L})-Bi\cdot I_0(2p\sqrt{L}) \qquad (7)$$

The resulting heat loss from the fin based on the one-dimensional analysis is

$$Q_{1A}=kA\frac{dT}{dx}|_{x=2L}=k\cdot\frac{2p}{\sqrt{2L}}\cdot\frac{C_1\cdot I_1(2p\sqrt{2L})-C_2\cdot K_1(2p\sqrt{2L})}{C_1\cdot I_0(2p\sqrt{2L})+C_2\cdot K_0(2p\sqrt{2L})} \qquad (8)$$

2.2 Two-dimensional analyses

2. 2. 1 Analyitical method (2A)

For geometry illustrated in Fig. 2, the two-dimensional governing partial differential equation is

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=0 \qquad (9)$$

Three boundary conditions and one energy balance equation are required to complete the formulation of the problem. These conditions are shown as Eqs. (10)-(13).

$$\theta=1 \qquad \text{at } x=0 \qquad -1\le y\le 1 \qquad (10)$$

$$\frac{\partial\theta}{\partial y}=0 \qquad \text{at } y=0 \qquad 0\le x\le L \qquad (11)$$

$$\frac{\partial\theta}{\partial x}+Bi\cdot\theta=0 \quad \text{at } x=L \quad -\frac{1}{2}\le y\le\frac{1}{2} \qquad (12)$$

$$-\int_0^1\left[\frac{\partial\theta}{\partial x}\right]_{x=0}dy=Bi\cdot\sqrt{4L^2+1}\int_{\frac{1}{2}}^1\theta\ dy-\int_0^{\frac{1}{2}}\left[\frac{\partial\theta}{\partial x}\right]_{x=L}dy \qquad (13)$$

The two-dimensional temperature profile, $\theta_{2A}(x, y)$, can be obtained by solving Eq. (9) with three boundary conditions as Eqs. (10)-(12) and one energy balance equation as Eq. (13) and the result is presented as Eqs. (14)-(21):

$$\theta_{2A}(x, y)=\sum_{n=1}^{\infty}N_n\cdot f(x)\cdot\cos(\lambda_n y) \qquad (14)$$

where

$$f(x)=\cosh(\lambda_n x)+f_n\cdot\sinh(\lambda_n x) \qquad (15)$$

$$f_n=-\frac{\lambda_n\cdot\tanh(\lambda_n L)+Bi}{\lambda_n+Bi\cdot\tanh(\lambda_n L)} \qquad (16)$$

$$=\frac{2Bi\cdot L\cdot(\lambda_n^2\cdot A_n+Bi\cdot\lambda_n\cdot B_n)}{\lambda_n^2\cdot\sqrt{1+4L^2}}+Bi\cdot\sin(\frac{\lambda_n}{2})-\sin(\lambda_n)\cdot C_n \qquad (17)$$

$$N_n=\frac{4\sin(\lambda_n)}{2\lambda_n+\sin(2\lambda_n)} \qquad (18)$$

$$A_n=\sinh(\lambda_n L)\cdot\cos(\lambda_n)+\frac{\cosh(\lambda_n L)\cdot\sin(\lambda_n)-\sin(\frac{\lambda_n}{2})}{2L} \qquad (19)$$

$$B_n=\cosh(\lambda_n L)\cdot\cos(\lambda_n)+\frac{\sinh(\lambda_n L)\cdot\sin(\lambda_n)}{2L}-\cos(\frac{\lambda_n}{2}) \qquad (20)$$

$$C_n=\lambda_n\cdot\sinh(\lambda_n L)+Bi\cdot\cosh(\lambda_n L) \qquad (21)$$

To obtain the eigenvalues, a forced analytical method (Kang and Look, 1993) is used. In this method, the eigenvalue, λ_1, is calculated using Eqs. (16) and (17); then the rest of the eigenvalues (λ_n($n=2, 3, 4, \cdots\cdots$)) are obtained by using Eq. (23) which is derived from Eq. (22). The direct application of a orthogonal principle used in the separation of variable method yields Eq. (22):

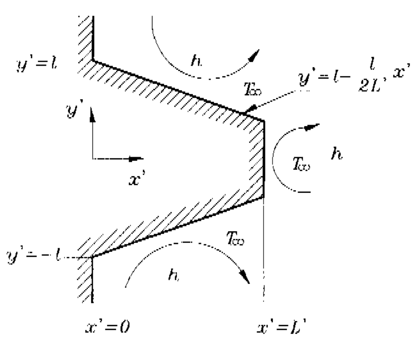

Fig. 2 Trapezoidal fin geometry used in the two-dimensional analysis.

$$\int_0^1 \cos(\lambda_1 y)\cos(\lambda_n y)\,dy = 0 \tag{22}$$

$$\lambda_n = (2\lambda_1 + \lambda_n) - 2(\lambda_1 + \lambda_n)\frac{\tan(\lambda_n)}{\tan(\lambda_1) + \tan(\lambda_n)} \tag{23}$$

The heat loss resulting from this two-dimensional analysis is given as

$$Q_{2A} = \int_{-1}^{1} -k\left[\frac{\partial\theta}{\partial x}\right]_{x=0} dy = 2k\sum_{n=1}^{\infty}\sin(\lambda_n)\cdot f_n \cdot N_n \tag{24}$$

2. 2. 2 Finite difference method (2F)

A two-dimensional finite difference method, presented in Kang and Look (1993), was based on the nodal arrangement of Fig. 3. Forty equations were solved simultaneously to obtain the value of the temperature at each node. Examples of the equations used are given by Eqs. (25)-(30).

For node 1 (and a similar form for the line points-11, 20 and 28)

$$1 - 2(1 + r^2)\cdot\theta_1 + 2r^2\cdot\theta_2 + \theta_{11} = 0 \tag{25}$$

For node 2 (and a similar form for all other interior points-3, 12 and so on)

$$1 + r^2\cdot\theta_1 - 2(r^2 + 1)\cdot\theta_2 + r^2\cdot\theta_3 + \theta_{12} = 0 \tag{26}$$

For node 10 (and a similar form for the points -19, 27 and 34)

$$1 + r^2\cdot\theta_9 - (1 + r^2 + Bi\cdot\Delta x\cdot\sqrt{r^2+1})\cdot\theta_{10} = 0 \tag{27}$$

For node 35

$$\theta_{28} - (1 + r^2 + Bi\cdot\Delta x)\cdot\theta_{35} + r^2\cdot\theta_{36} = 0 \tag{28}$$

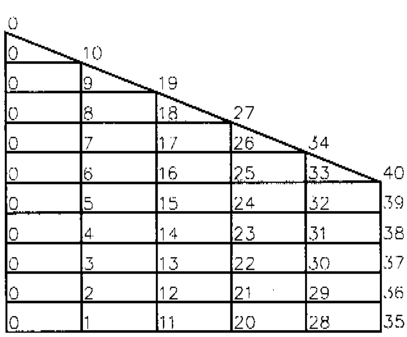

Fig. 3 Upper half fin geometry showing the 40 nodes in the finite difference method.

For node 36 (and a similar form for the points -37, 38 and 39)

$$\theta_{29} + \frac{r^2\cdot\theta_{35}}{2} - (1 + r^2 + Bi\cdot\Delta x)\cdot\theta_{36} + \frac{r^2\cdot\theta_{37}}{2} = 0 \tag{29}$$

For node 40

$$\theta_{33} + \frac{r^2\cdot\theta_{39}}{2} - \left(1 + \frac{r^2}{2} + \frac{Bi\cdot\Delta x}{2} + \frac{Bi\cdot\Delta x\cdot\sqrt{r^2+1}}{2}\right)\cdot\theta_{40} = 0 \tag{30}$$

The heat loss can be calculated for this two-dimensional analysis using Eq. (31).

$$Q_{2F} = k\cdot Bi\cdot\sqrt{(\Delta x)^2 + (\Delta y)^2}\cdot\left(\frac{1}{2} + \theta_{10} + \theta_{19} + \theta_{27} + \theta_{34} + \frac{1}{2}\theta_{40}\right) + k\cdot Bi\cdot\Delta y\cdot\left(\frac{1}{2}\theta_{35} + \theta_{36} + \theta_{37} + \theta_{38} + \theta_{39} + \frac{1}{2}\theta_{40}\right) \tag{31}$$

2. 2. 3 Modified finite difference method (MF)

For this method, the nodes near and on the upper surface are treated differently than in (2F). That is, the triangular shape (see Fig. 3) is utilized for the nodes near and on the upper fin surface while all the interior points were treated as components of the rectangular shape, as in Kang and Look (1993). So the equations for all the interior nodes are the same as the (2F) case just discussed. The modified equations for these nodes are shown as Eqs. (32)-(34).

For node 9 (and a similar form for the points -18, 26 and 33)

$$1 + r^2\cdot\theta_8 - \frac{13}{6}(1 + r^2)\,\theta_9 + \frac{7}{6}(r^2\cdot\theta_{10} + \theta_{19}) = 0 \tag{32}$$

For node 10 (and a similar form for the points -19, 27 and 34)

$$1 + r^2\cdot\theta_9 - \left(1 + r^2 + \frac{6}{7}Bi\cdot\Delta x\cdot\sqrt{r^2+1}\right)\theta_{10} = 0 \tag{33}$$

For node 40

$$\theta_{33} + \frac{3}{7}r^2\cdot\theta_{39} - \left\{1 + \frac{3}{7}r^2 + \frac{3}{7}Bi\cdot\Delta x\right.$$

$$\cdot\ (1+\sqrt{r^2+1})\Big\}\theta_{i0}=0 \tag{34}$$

Note that in comparing Eqs. (33) and (34) to Eqs. (27) and (30), they have the same forms but different coefficients. The equation for the heat loss for this modified finite difference method is also Eq. (31). Finally, for both of these finite difference methods, the non-dimensional fin length was restricted to be less than 2 in order to prevent errors which might occur due to large Δx values when the non-dimensional fin length increased.

3. Numerical Results and Discussions

Figure 4(a) presents the relative difference in the heat loss from the trapezoidal fin based on the two-dimensional analytical solution as a function of Biot number for L=1. This figure shows that the relative difference for 2F and MF increases while the relative difference for 1A first increases and then decreases as the Biot number increases from 0.1 to 1.0. It is noted that the relative difference is reduced by about a half by using the modified finite difference method instead of using the usual finite difference method. Results for the same configuration as in Fig. 4(a) except that L =2 are presented in Fig. 4(b). The trend of the curves for all three cases is somewhat similar but the relative difference for 1A varies from positive value to negative value as the Biot number varies from 0.1 to 1.0. The fact that the relative difference in the case of MF is smaller than that for the other two cases as Biot number varies from 0.1 to approximately 0.3 is shown in this figure.

Figures 5(a)-5(c) show the variation of the non-dimensional temperature along the non -dimensional x coordinate by using the four methods for the non-dimensional fin length, L= 1, and Biot number=0.01, 0.1, and 1.0. For the two-dimensional analyses, the non-dimensional temperature profiles are those along the fin center line (i. e. y=0). Note, because of the coordinate system differences, in order to compare the one -dimensional analytical method to the two -dimensional analyses directly, 1.8 is substituted into x for the one dimensional analysis as x=0.2 for the two-dimensional analyses and so on (see Fig. 1 and Fig. 2). The variations of the non -dimensional temperatures along the non-dimensional x coordinate for Bi=0.01 case are shown in Fig. 5(a). The differences in the values for each of the four methods increase with x. It also shows the value obtained from the two-dimensional analytical method is the highest while the value obtained from the one-dimensional analytical

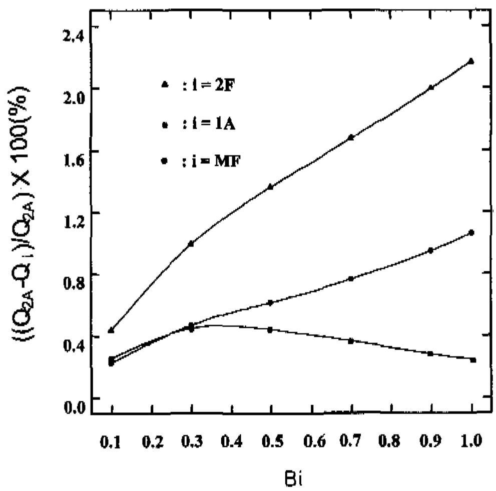

Fig. 4(a) The relative difference in the heat loss from the trapezoidal fin compared to the two-dimensional analytical method as a function of Biot number for L=1.

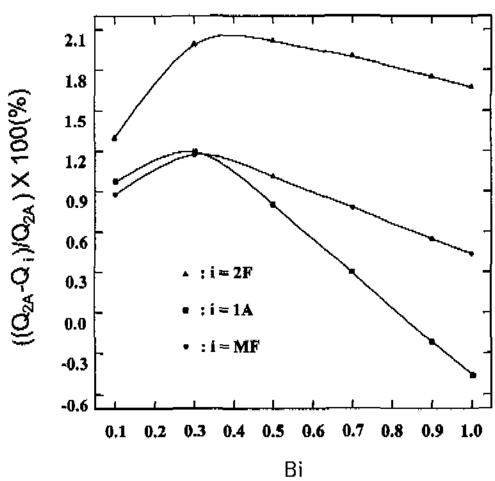

Fig. 4(b) The relative difference in the heat loss from the trapezoidal fin compared to the two-dimensional analytical method as a function of Biot number for L=2.

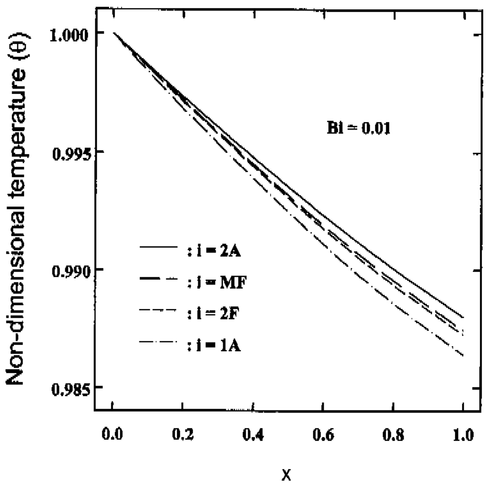

Fig. 5(a) The variation of the non-dimensional temperature at the fin center line (in the cases of two-dimensional analyses) along the x coordinate for L=1 and Bi=0.01.

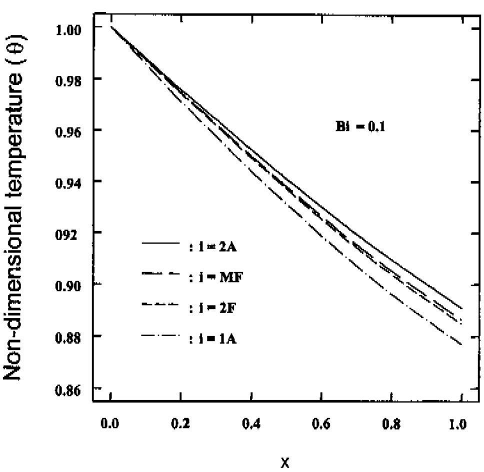

Fig. 5(b) The variation of the non-dimensional temperature at the fin center line (in the cases of two-dimensional analyses) along the x coordinate for L=1 and Bi=0.1.

method is the lowest. The same trend as in Fig. 5 (a) but with large profiles slope are shown in Fig. 5(b). Figure 5(c) depicts the same type of information as was presented as Figs. 5(a)-5(b) but for Bi=1.0. Most notable in this figure is the value for MF is almost the same as the values for 2F and these values are close to the value for 2A. These figures also show the values calculated by the one-dimensional analytical method disagrees markedly from the three other methods as x

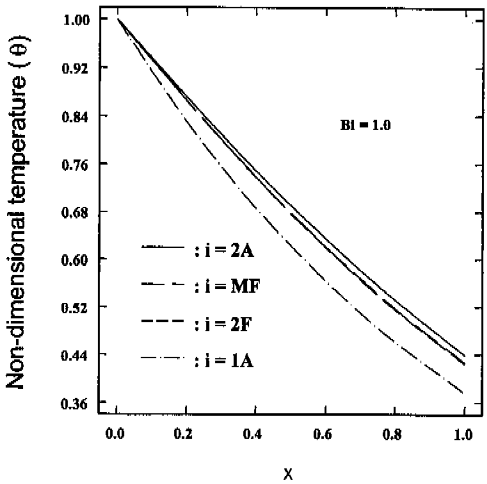

Fig. 5(c) The variation of the non-dimensional temperature at the fin center line (in the cases of two-dimensional analyses) along the x coordinate for L=1 and Bi=1.0.

increases (see also Table 1).

Table 1 lists the relative difference in three other methods of computing the non-dimensional temperature as compared to a two-dimensional analytical method for L=1 and Bi=0.01, 0.1, 1.0. This table shows the relative difference for MF is smallest, in general. In the case of 1A, the relative difference is good for Bi=0.01 and 0.1 but this relative difference is over 14% along the center line of the fin and varies from -8.64% to 7.77% along the upper surface of the trapezoidal fin for Bi=1.0.

Figure 6(a) presents the relative differences in the heat loss from the trapezoidal fin using the other methods as compared to the two-dimensional analytical method as a function of L for Bi =0.01 case. This relative difference increases as L increases for each of three methods but they are all less than 0.2% for the given range of the non-dimensional fin length. It is noted that the relative difference for MF is almost zero until the non-dimensional fin length reaches 0.5. The same description but for Bi=0.1 case is shown in Fig. 6(b). On the whole, the relative differences for Bi =0.1 are larger than those for Bi=0.01 but the maximum value occurred for 2F when L=2.0 is still less than 1.3%. This figure also shows that the relative error for MF is still near zero for $L \leq 0.5$

Table 1 Relative errors in three different methods of computing the temperature as compared to a two-dimensional analytical method for L =1.

	x	i=MF	i=2F	i=1A
		$((\theta_{2A}-\theta_i)/\theta_{2A})\times100$ (%)		
		Along the center line of the fin		
Bi=0.01	0.2	0.0147	0.0195	0.0488
	0.4	0.0295	0.0388	0.0894
	0.6	0.0431	0.0569	0.1240
	0.8	0.0532	0.0707	0.1504
	1.0	0.0569	0.0761	0.1619
Bi=0.1	0.2	0.1267	0.1643	0.4780
	0.4	0.2577	0.3338	0.8723
	0.6	0.3827	0.4964	1.2043
	0.8	0.4770	0.6234	1.4591
	1.0	0.5116	0.6743	1.5707
Bi=1.0	0.2	0.6233	0.6751	4.6428
	0.4	1.3406	1.4872	8.3055
	0.6	2.1183	2.3646	11.2122
	0.8	2.7511	3.1168	13.3543
	1.0	2.9525	3.3993	14.2887
	x	Along the upper lateral surface of the fin		
Bi=0.01	0.2	-0.0328	-0.0988	-0.0694
	0.4	0.0021	0.0231	0.0445
	0.6	0.0436	0.0704	0.0038
	0.8	0.0774	0.1099	0.0555
	1.0	0.0828	0.1211	0.0951
Bi=0.1	0.2	-0.3050	-0.1430	-0.7247
	0.4	0.0201	0.2217	-0.4892
	0.6	0.4015	0.6450	-0.0169
	0.8	0.7137	1.0065	0.4958
	1.0	0.7570	1.1055	0.8936
Bi=1.0	0.2	-2.0383	-0.7157	-8.6424
	0.4	0.8528	2.1954	-6.1134
	0.6	3.5339	4.8999	-1.2775
	0.8	5.3752	6.8840	3.8040
	1.0	5.0297	6.8979	7.7768

and the relative difference for 1A is somewhat close to that for MF. Figure 6(c) depicts the same

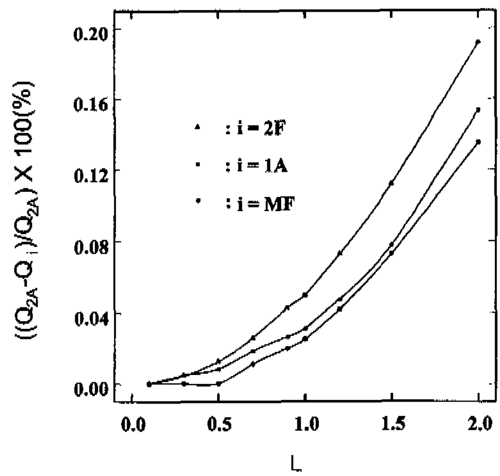

Fig. 6(a) The relative error in the heat loss from the trapezoidal fin compared to the two-dimensional analytical method as a function of L for Bi=0.01.

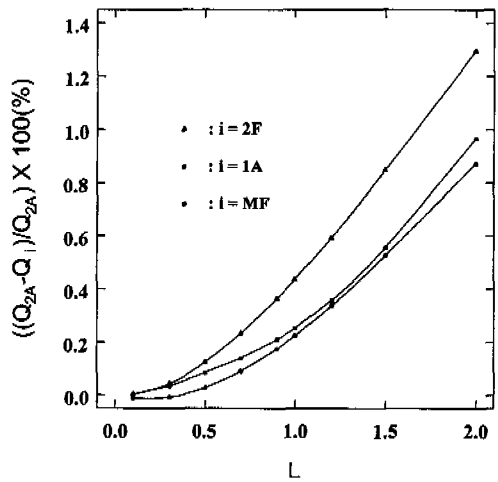

Fig. 6(b) The relative error in the heat loss from the trapezoidal fin compared to the two-dimensional analytical method as a function of L for Bi=0.1.

setup as Figs. 6(a)-6(b) but for Bi=1.0. In this case, the relative difference varies dramatically and irregularly especially for 1A as L increases from 0.1 to 2.0. In this figure, the relative difference for 1A varies from approximately 0.8 to -0.5% while the relative difference for 2F varies from 0.1 to 2.4% approximately.

Figure 7 presents the variation of the non-dimensional heat loss from the trapezoidal fin as a function of the non-dimensional fin length for

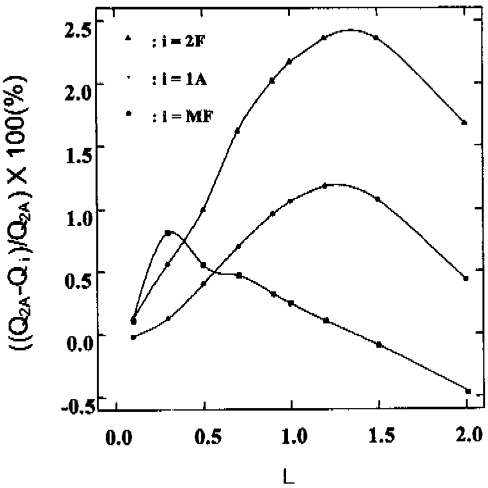

Fig. 6(c) The relative error in the heat loss from the trapezoidal fin compared to the two-dimensional analytical method as a function of L for Bi = 1.0.

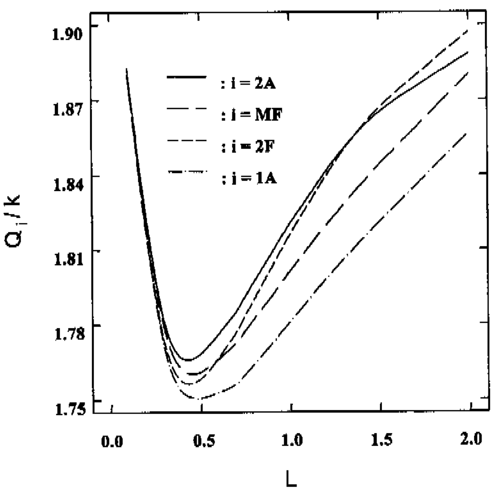

Fig. 7 The variation of the non-dimensional heat loss from the trapezoidal fin as a function of L for Bi = 1.0.

Bi = 1.0 for four methods. For all four methods, the heat loss decreases as L increases until 0.5 and then the heat loss increases as L varies from 0.5 to 2.0. It can be noted that the trend of the variation of the heat loss as a function of the non-dimensional fin length is similar for all four methods even though the relative difference for the three methods in case of Bi = 1.0 vary somewhat irregularly as L varies from 0.1 to 2.0 as shown in Fig. 6(c).

4. Conclusions

This paper describes four methods of approach to solving a heat transfer problem involving a trapezoidal fin. The following conclusions can be made from the results and it is noted that some are consistent with conventional wisdom:

(1) The relative differences in the heat loss from the trapezoidal fin using the three methods as compared to the two-dimensional analytical method as a function of Biot number and as a function of the non-dimensional fin length vary somewhat irregularly. Even so, they can be used for analysing a trapezoidal fin and the results are in agreement with each other within 3% error for the given range of Biot numbers.

(2) In the view of the non-dimensional temperature, the one-dimensional analytical method does not show good result as compared to other three methods when Biot number is 1.0.

(3) By using the modified finite difference method instead of the finite difference method which was used in Kang and Look (1993), the relative difference as compared to a two dimensional analytical method is reduced considerably based on the heat loss.

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